Logical Quantifications: Conversions

$R(x)$: $x \in$ 3342_class

$P(x)$: x receives A+

$$(\forall X \bullet R(X) \Rightarrow P(X)) \Leftrightarrow \neg(\exists X \bullet R \wedge \neg P)$$

$$(\exists X \bullet R \wedge P) \Leftrightarrow \neg(\forall X \bullet R \Rightarrow \neg P)$$

De Morgan

Relating Sets: Exercises

Sets: Exercises

Set membership: Rewrite $e \notin S$ in terms of $\in$ and $\neg$

Find a common pattern for defining:

1. $=$ (numerical equality) via $\leq$ and $\geq$
2. $=$ (set equality) via $\subseteq$ and $\supseteq$

$S = \{1, 2, 3\}$, $T = \{2, 3, 1\}$, $U = \{3, 2\}$

	S		T		U	
S	$\subseteq$	$\subset$	$\subseteq$	$\subset$	$\subseteq$	$\subset$
T	$\subseteq$	$\subset$	$\subseteq$	$\subset$	$\subseteq$	$\subset$
U	$\subseteq$	$\subset$	$\subseteq$	$\subset$	$\subseteq$	$\subset$

Is set difference ($\setminus$) commutative?

Exercise:

How many **sets** of size 3 can be made out of values 1, 2, 3, 4, 5?

Combinations: Formula and Interpretation

Power Set

Calculate the power set of $\{1, 2, 3\}$.

Given a set S, formulate the cardinality of its power set.

Cardinality of Power Set: Interpreting Formula

- Calculate by considering subsets of various cardinalities.
- Calculate by considering whether a member should be included.

Set of Tuples

Given n sets $S_1, S_2, \ldots, S_n$, a ***cross/Cartesian product*** of theses sets is a set of n-tuples.

Each ***n-tuple*** $(e_1, e_2, \ldots, e_n)$ contains n elements, each of which a member of the corresponding set.

$$S_1 \times S_2 \times \cdots \times S_n = \{(e_1, e_2, \ldots, e_n) \mid e_i \in S_i \land 1 \leq i \leq n\}$$

Example: Calculate {a, b} ✘ {2, 4} ✘ {$, &}

Set of Possible Relations

- Set of possible relations on S and T:
- Dedicated symbol for set of possible relations on S and T:
- Declare that set r is a relation on S and T:

Example: Enumerate **all** relations on {a, b} and {2, 4}.

Hint: How many?

Relational Operations: Domain, Range, Inverse

$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$

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Exercise: Relate the domains and ranges of r and its inverse.

Relational Operations: Image

r = {(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)}

Exercises

- **Image** of {a, b} on r?
- **Image** of {1, 2} on r?
- **Image** of {1, 2} on the **inverse** of r?
- Calculate r's **range** via an **image**.
- Calculate r's **domain** via an **image**.

Relational Operations: Restrictions vs. Subtractions

r = {(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)}

r = {(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)}

r = {(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)}

r = {(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)}